Frequency locking, quasiperiodicity, and chaos in modulated self-pulsing semiconductor lasers

H. G. Winful, Y. C. Chen, and J. M. Liu
GTE Laboratories, Inc., 40 Sylvan Road, Waltham, Massachusetts 02254

(Received 5 November 1985; accepted for publication 13 January 1986)

A transition to chaos via quasiperiodicity is observed in the output of a directly modulated self-pulsing semiconductor laser. By sweeping the frequency and amplitude of the current modulation, several frequency-locked states (Arnol'd tongues) are mapped out directly. Good agreement with the predictions of a rate equation model is obtained.

In this letter, experimental results on the quasiperiodic route to chaos in a modulated self-pulsing semiconductor laser are presented for the first time. A direct determination of the structure of the frequency-locked states is also reported. The results are in good agreement with the predictions of a rate equation model for a laser with a saturable absorber.

The observed behavior of the driven self-oscillatory laser has features that are generic to nonlinear systems characterized by the presence of two competing frequencies.[1] If the driving frequency is an integer multiple or submultiple of the intrinsic pulsation frequency the phenomenon of frequency locking can occur. Such frequency locking is of practical importance in short pulse generation and in the stabilization of oscillators.[2] If the two frequencies are incommensurate, the resulting oscillations are usually quasiperiodic. A direct transition from quasiperiodic to chaotic oscillation can occur if the amplitude of the external modulation is increased while maintaining a fixed irrational value for the ratio between the two frequencies. Certain universal features of this transition to chaos have recently been tested in a fluid dynamical experiment.[3] Here we report this transition in an optical system that is well characterized by a finite-dimensional set of rate equations.

The small-signal modulation response of semiconductor lasers exhibits a peak at a relaxation oscillation frequency that characterizes the rate of energy exchange between photons and charge carriers within the cavity. In stable, well-behaved lasers, this resonance is a rather broad, flat feature. By momentarily pulsing the drive current beyond the threshold for catastrophic optical damage, self-sustained pulsations can be induced in a nominally stable laser. These pulsations arise because of the formation of defects that act as saturable absorbers and lead to repetitive Q switching.[4] The spectrum of the pulsing laser now exhibits sharp peaks at the intrinsic oscillation frequency and its harmonics. For the AlGaAs/GaAs lasers used in our experiments, the fundamental pulsation frequency f_0 can be tuned between 0.5 and 3 GHz by varying the dc bias current.

The second frequency $f_{\rm ext}$ is imposed by an rf generator whose output modulates the laser pumping current at rates between 0.3 and 2.0 GHz. In the presence of the external modulation, the intrinsic resonance frequency is shifted by an amount that depends on the amplitude of the modulation. We thus speak of a "dressed" intrinsic frequency f_0' and define a winding number $\rho = f_0'/f_{\rm ext}$. If the winding number takes on a rational value p/q, where p and q are integers, the output pulsation frequency locks to a harmonic or subharmonic of the modulation. There is a range of frequency detunings within which the external modulation can effectively entrain the self-pulsation frequency. This locking range increases with the amplitude of the external modulation. By sweeping the frequency and amplitude of the modulation, we have mapped out the structure of the frequency-locked states. On a plot of modulation amplitude versus frequency ratio, the locked states form regions known as Arnol'd tongues[5] whose boundaries separate the periodic motions from the quasiperiodic and aperiodic oscillations. Figure 1 shows several of these frequency-locked regions for small integer values of p and q. The organization of the locking regions follows the Farey tree structure.[6] Between any two locked bands with winding numbers p_1/q_1 and p_2/q_2, there exists another locked band whose winding number is given by the Farey sum $p_3/q_3 = (p_1 + p_2)/(q_1 + q_2)$. For example, the locked band with winding number 2/3 is the Farey composition of the bands 1/1 and 1/2. Locked states with large denominators have very narrow widths, are easily destabilized by noise, and are thus difficult to resolve. There

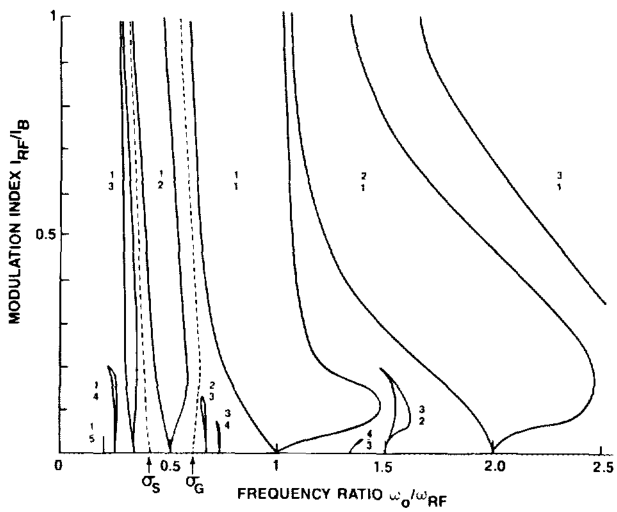

FIG. 1. Frequency-locked regions (Arnol'd tongues) for a modulated self-pulsing laser. The plot shows the modulation depth vs the ratio of the intrinsic pulsation frequency at zero drive (ω_0) to the external modulation frequency. The dotted lines show paths of fixed winding number at the golden mean σ_g and at the silver mean σ_s. Because the intrinsic frequency shifts with the amplitude of the modulation, paths of fixed winding number are not straight lines.

are other locked states (such as $\rho = 4/5$) which by virtue of their proximity to the strong fundamental resonance are pulled by that resonance and tend to merge with it. We note that the measured Arnol'd tongue structure is independent of the direction in which the frequency or amplitude of the modulation is varied. No sign of hysteresis is observed.

Outside the locking regions, the winding number ρ is irrational. For small modulation amplitudes, the oscillations in the unlocked regions are generally quasiperiodic, reflecting a complex beating between the two incommensurate frequencies. The time series and power spectra associated with the quasiperiodic oscillations are shown in Fig. 2 for a modulation index of $m = 0.05$. The quasiperiodic spectra (dashed curves) contain discrete lines at the modulation frequency, the intrinsic resonance frequency, and its harmonics, as well as the various combination frequencies.

By maintaining a fixed irrational value (to within 1.0%) for the winding number, we have observed a direct transition from quasiperiodicity to chaos without frequency locking. This was accomplished by slowly increasing the depth of modulation and adjusting the driving frequency to maintain a constant ratio between that frequency and the shifted intrinsic resonance frequency. The chosen irrational values of the winding number were the golden mean $\sigma_g = (\sqrt{5} - 1)/2$ and the silver mean, $\sigma_s = \sqrt{2} - 1$. These represent the "worst" irrational numbers in the sense that they are the most difficult to approximate by rational numbers. By staying close to these values, frequency locking is avoided and universal features of the transition from quasiperiodicity to chaos can be studied. The chaotic time series and spectra thus obtained are also shown in Fig. 2. At the transition to chaos the sharp frequency peaks in the spectrum disappear. There is a significant rise in the background and the spectrum now consists of a broad, continuous distribution with a peak at the modulation frequency. On close examination of some of the single-shot chaotic time series, it is possible to identify small time intervals that bear the signature of nearby resonances. The oscillating system appears to wander erratically between several phase-locked resonances. It is believed that most driven nonlinear oscillators proceed to chaos via such an interaction of resonances.[1]

The observed phenomena of self-pulsing, quasiperiodicity, frequency locking, and chaos have been successfully modeled by rate equations for a laser with a fast saturable absorber[7]:

$$\frac{dN}{dt} = I - N - G\left(N - \frac{1}{2}\right)S, \tag{1}$$

$$\frac{dS}{dt} = G\left(N - \frac{1}{2}\right)S - \frac{S}{\tau} + \beta N - hS\left(1 + \frac{S}{S_0}\right)^{-1}. \tag{2}$$

Here N and S are the carrier and photon densities, normalized by the threshold carrier density. The modulated injection current, normalized by its value at threshold, is

$$I = I_b + I_{rf}\sin(\Omega t). \tag{3}$$

Time is measured in units of the spontaneous electron lifetime and τ is the photon lifetime in those units. G is proportional to the gain, β is the fraction of spontaneous emission coupled into the lasing mode, and h characterizes the amount of saturable absorption. The nonlinear absorber bleaches at a saturation photon density S_0. Note that modulation of the laser injection current makes the system of Eqs. (1) and (2) nonautonomous[8] and provides the third degree of freedom required for chaos.

Figure 3 shows an example of the quasiperiodic, frequency-locked, and chaotic output spectra computed from Eqs. (1)–(3). The numerical solutions confirm the deterministic nature of the observed random behavior. Furthermore, they show that the simple period-doubling route to

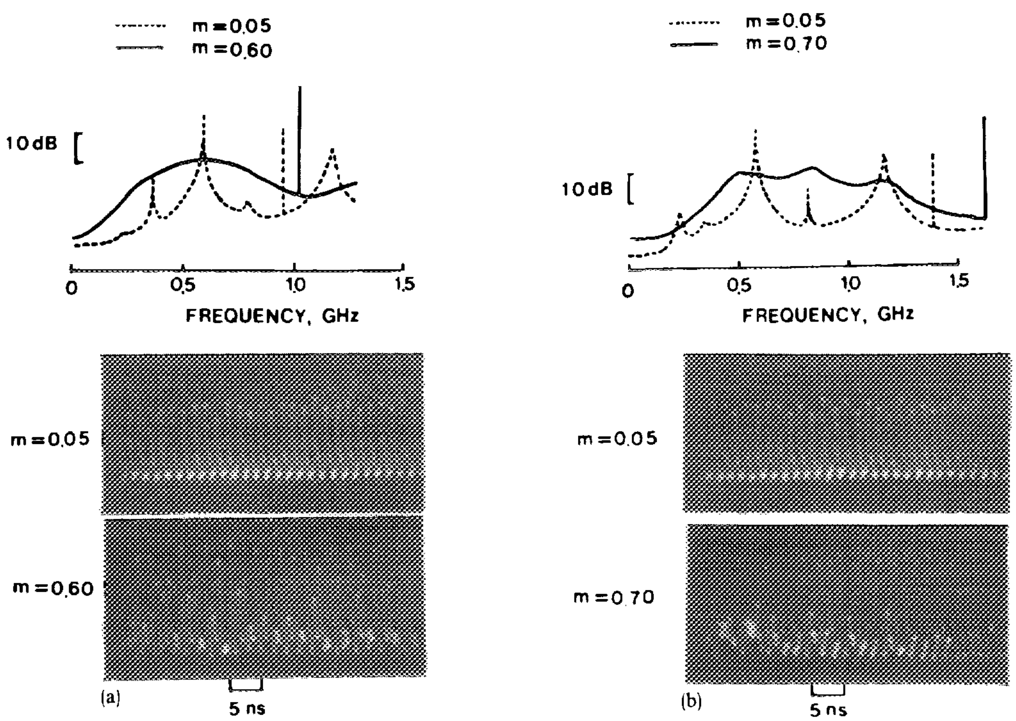

FIG. 2. Spectra of quasiperiodic oscillations (dashed lines) and chaotic oscillations (solid lines) and the associated time series obtained for fixed winding numbers near (a) the golden mean and (b) the silver mean. m is the modulation index. The modulation frequency is ~1 GHz in (a) and ~1.5 GHz in (b).

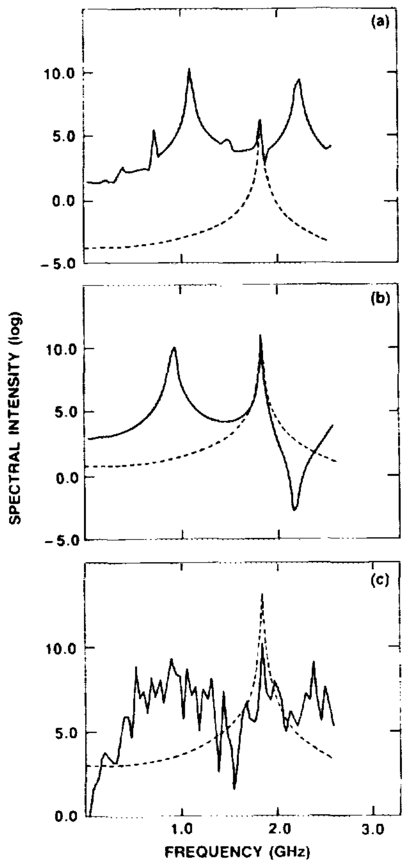

FIG. 3. Computed output intensity spectra for a frequency ratio $\omega_0/\omega_{ext} = 0.59$ and different modulation depths, m. The external modulation is shown as the dashed line. (a) $m = 0.01$; quasiperiodic (b) $m = 0.1$; phase-locked (c) $m = 0.3$; chaotic. Note that in this sequence the winding number is not fixed. The parameters used in Eqs. (1)–(3) are $I = 1.2$, $G = 3.0\times10^3$, $\tau = 3\times10^{-4}$, $\beta = 5.0\times10^{-5}$, $h = 240$, $S_0 = 0.001$. The spontaneous carrier lifetime is 3.0 ns.

chaos previously suggested[9] for modulated self-pulsing lasers is an unlikely one. Over much of parameter space frequency locking and quasiperiodic oscillations are observed. The simulations also reveal the existence of high-period locked states which, because of intrinsic noise, are not observed experimentally.[10] We note that quasiperiodic and frequency-locked dynamics are absent in stable lasers that do not exhibit self-excited oscillations. In such lasers, the response to a step input current consists of highly damped relaxation oscillations. The predicted response to a current modulation is then either periodic or chaotic, with the transition to chaos occurring via a series of successive subharmonic bifurcations.[11]

In conclusion, we have experimentally determined the structure of the frequency-locked regimes of a modulated, self-pulsing laser. By maintaining a fixed irrational winding number, we have observed a direct transition from quasiperiodicity to chaos in these lasers. Our results show that the period-doubling route to chaos is not the only one for modulated semiconductor lasers. In fact, for lasers with well-developed pulsations, the route to chaos via quasiperiodicity is the more likely one.

One of us (H. G. W.) acknowledges illuminating discussions with Professor F. T. Arecchi.

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