when the beam intensities were reduced.

(2) The nanosecond pulse was not bandwidth limited, implying that it had significant amplitude modulation. Since the picosecond pulse was not long enough to average these fluctuations, it may have seen pulse-to-pulse intensity variations in the temporal overlap with the nanosecond pulse that were larger than are implied by the total pulse-to-pulse energy variations. Since the FIR energy was relatively insensitive to the nanosecond energy, these modulations would have to be very large.

Neither of these explanations is entirely satisfactory, but the presence of occasional high-efficiency pulses can be seen as encouraging. Their presence implies that a fundamental limit to conversion efficiency has not yet been reached. If the source of these fluctuations can be understood, it may be possible to consistently generate FIR pulses with the efficiencies which are now seen only erratically. Even without this improvement, intense, continuously tunable, narrow-bandwidth FIR pulses have been produced which can be a valuable spectroscopic tool.

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Subharmonic bifurcations and irregular pulsing behavior of modulated semiconductor lasers

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We report the observation of subharmonic bifurcations and irregular pulsing behavior in the output of directly modulated diode lasers. The bifurcation sequence differs from the predicted scenarios most noticeably in the presence of large intrinsic fluctuations which impede any stable period-two oscillations for non-self-pulsing lasers and period-four oscillations for self-pulsing lasers. The nondeterministic noise makes it difficult to observe high-period bifurcation and chaotic behavior in semiconductor lasers by direct current modulation.

The problem of existence of lasing regimes characterized by an irregular response of a semiconductor laser to a periodic modulation of the injection current has been of considerable interest recently. Based on coupled nonlinear rate equations for carrier and photon densities, Lee *et al.*[1] predicted a period doubling route to chaos in a directly modulated semiconductor laser as the modulation index increases. Kawaguchi[2] showed a similar route to chaos in the optical resposne of a directly modulated, tandem-type semiconductor laser in which one of the sections acts as a saturable absorber. While chaotic behavior has been experimentally observed in other lasers with modulated parameters,[3,4] chaos in a modulated diode laser still remains to be demonstrated. In this letter we report the observation of subharmonic bifurcations and irregular pulsing behavior in the output of semiconductor lasers subjected to deep sinusoidal modulation of the injection current. The experimentally measured bifurcation sequences differ from the predicted scenarios in the presence of large intrinsic laser amplitude fluctuations which impede the bifurcation into stable period-two oscillations for lasers without self-sustained pulsations and period-four oscillations for lasers exhibiting self-sustained pulsations. We find that the intrinsic fluctuations make it difficult to observe high-period bifurcation and chaotic behavior in

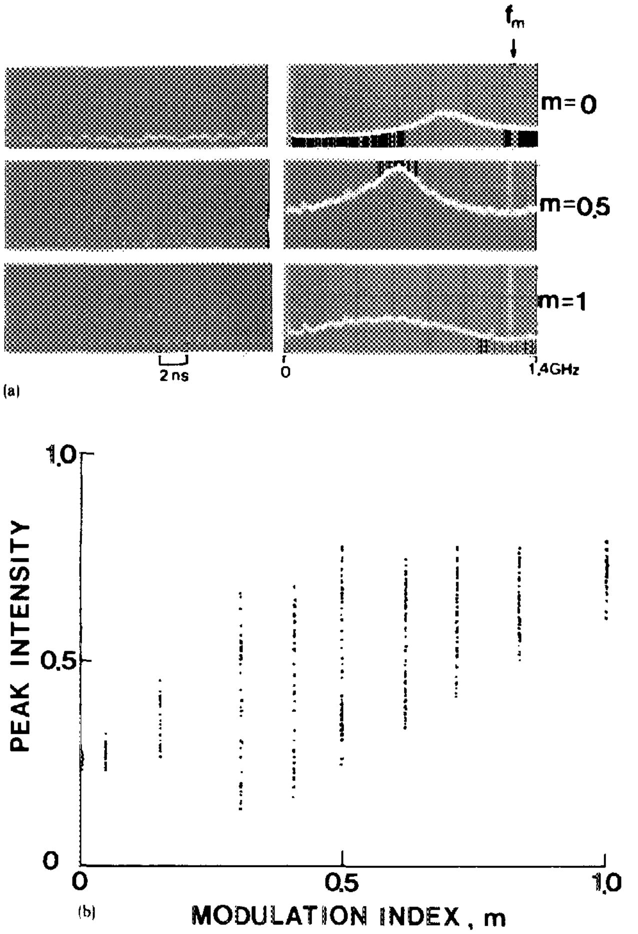

FIG. 1. (a) Oscilloscope traces and frequency spectra of the optical response of a directly modulated non-self-pulsing transverse-junction-stripe laser at different modulation indices. (b) Bifurcation diagram of peak photon density vs modulation index.

semiconductor lasers by direct current modulation.

We have studied a variety of index-guided and gain-guided AlGaAs/GaAs lasers exhibiting linear output characteristics and no self-sustained pulsations, and a number of lasers exhibiting self-sustained pulsations induced by optical damage. The lasers are biased at a dc current, typically at $I_B = 1.1I_{\mathrm{th}}$, so that the intrinsic resonance frequency is below the modulation frequency. To facilitate the observation of the irregular response in real time, the modualtion frequency is chosen to be $f_m = 1.2$ GHz, the upper limit of the fast oscilloscope. The laser output is detected by an AEG Telefuken S171P avalanche photodiode with a bandwidth over 3 GHz. The waveform of the detector is displayed on a Tektronix 7104 oscilloscope operated in the single sweep mode. The frequency spectra are simultaneously monitored using an rf spectrum analyzer.

Figure 1(a) shows the oscilloscope traces and the frequency spectra of the optical response of a non-self-pulsing transverse-junction-stripe laser at different values of the modulation index $m = I_{\mathrm{rf}}/I_B$. The bifurcation diagram of the randomly sampled peak photon density versus modulation index of this laser is shown in Fig. 1(b). In the absence of the rf modulation, the laser output is characterized by intrinsic fluctuations whose frequency is centered around the relaxation oscillation frequency. The fluctuations increase in amplitude and the waveform evolves into an intermittent period-two pattern as the modulation index increases. In the frequency domain, as viewed with the spectrum analyzer, the evolution into the pattern of intermittent period-two corresponds to a gradual pulling of the intrinsic resonance toward $f_m/2$, the subharmonic of the modulation frequency. With further increase in modulation index, there is no further bifurcation and the oscillation gradually becomes period-one with reduced fluctuation. The general features shown in Fig. 1(b) are shared by non-self-pulsing lasers of different structures, including the channeled-substrate-planar structure lasers, the buried-heterostructure lasers, and the oxide-defined narrow-stripe (4 μm) lasers. We note that the fluctuations in the optical response of the narrow-stripe lasers are smaller and the splitting in the distribution of the peak photon density near $m = 0.5$ in the bifurcation diagram is not clearly resolved. On the other hand, in lasers exhibiting strong intrinsic noise, it is possible to produce intermittent period-two oscillations with a clear splitting in the distribution of peak photon density by controlling both the dc bias and the rf modulation index.

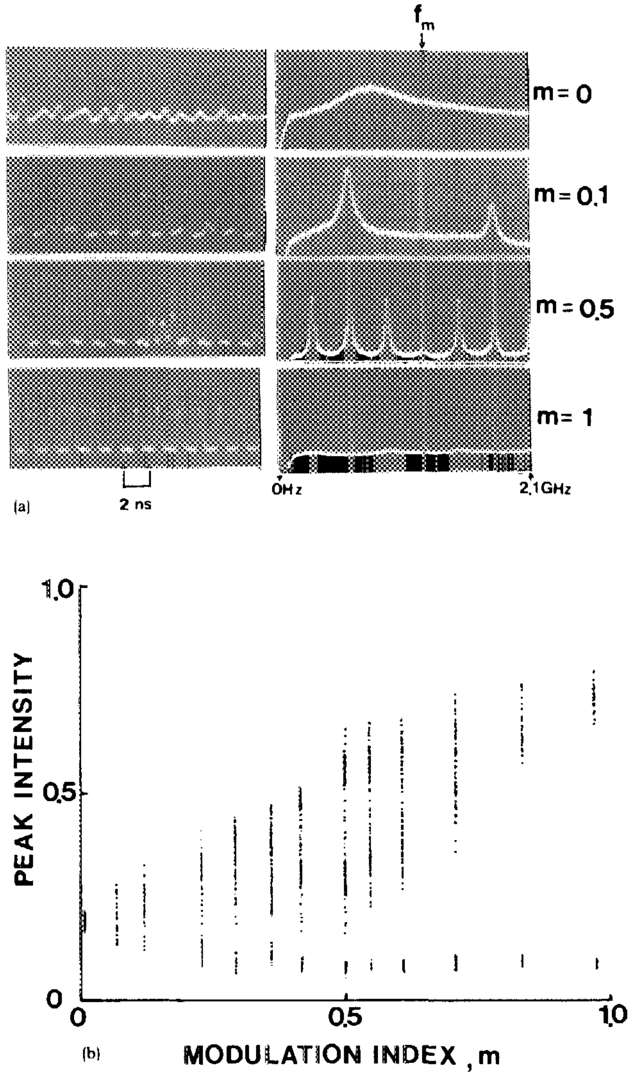

FIG. 2. (a) Oscilloscope traces and frequency spectra of the optical response of a directly modulated self-pulsing transverse-junction-stripe laser at different modulation indices. (b) Bifurcation diagram of peak photon density vs modulation index.

Figure 2(a) shows the oscilloscope traces and the frequency spectra of the optical response of a self-pulsing trans-

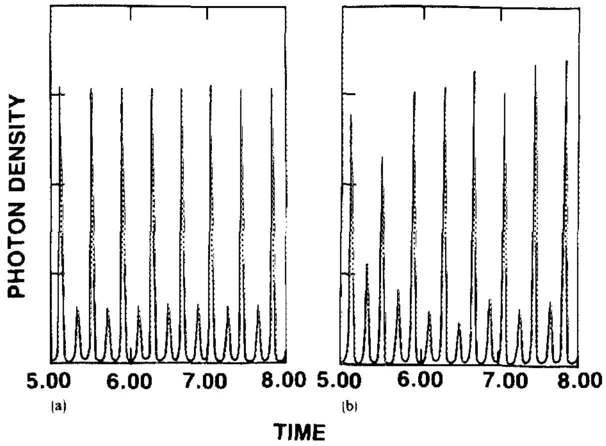

FIG. 3. (a) Photon density vs time for period-two solution of rate equations: (a) without noise and (b) with Langevin noise sources.

verse-junction-stripe laser at different values of modulation index. The bifurcation diagram of this laser is shown in Fig. 2(b). For this study, the dc bias current is adjusted so that the intrinsic resonance frequency is near one-half the modulation frequency. At this bias level, the intrinsic resonance is still rather broad because the self-sustained pulsation is not fully developed. The application of small rf current modulation produces period-two oscillations with large fluctuations. In the frequency domain, this corresponds to the formation of subharmonic resonance at $f_m/2$. The subharmonic resonance sharpens at $m = 0.22$. At this point, the laser output consists of pulses of alternating heights, as previously reported,[5,6] resulting in a splitting in the distribution of peak photon density in the bifurcation diagram. With further increase in modulation index, the period-two pattern evolves into an intermittent period-four, resulting in further splitting in the upper branch of the bifurcation diagram near $m = 0.5$. In the frequency domain, this corresponds to the emergence of a series of broader resonance peaks at $(2n + 1) f_m/4$, with $n = 0,1,2,\ldots$ With further increase in modulation index, there is no further bifurcation and the oscillation gradually becomes period-two with reduced fluctuations. An exception is found when the intrinsic resonance is near one-third of the modulation frequency. Over a small range of modulation index, subharmonic resonance peaks at $f_m/3$ and $f_m/6$ are observed.

The experimentally observed bifurcation behavior differs from the predicted bifurcation sequence in the presence of large intrinsic fluctuations for all modualtion indices, and the absence of stable high-period bifurcations. In non-self-pulsing lasers, we have observed no more than an intermittent period-two while in self-pulsing lasers, we have observed no more than an intermittent period-four. The discrepancy between the previous theoretical and experimental results exists because the previous analyses were based on numerical solutions of the deterministic noise-free rate equations that govern the interaction between electrons and photons in the laser. However, experimental data show that the output of a cw diode laser exhibits large amplitude fluctuations with a frequency centered at the relaxation oscillation resonance. These fluctuations arise from the quantum nature of spontaneous emission and cannot be eliminated in real diode lasers. In effect, the laser acts as a noise-driven relaxation oscillator. When the intrinsic fluctuation is comparable to or larger than the fine amplitude variations that characterize high-period bifurcations, the presence of the fluctuation will effectively truncate the bifurcation sequence. (Such a truncation of the bifurcation sequence has been shown to occur in a driven anharmonic oscillator perturbed by noise.[7]) The irregular pulsation in the region between the intermittent period-four and the final period-two may be a result of the chaotic behavior mixed with intrinsic noise.

In order to model the bifurcation behavior of modulated diode lasers more realistically, we have included random Langevin noise sources in the standard laser rate equations.[8] The noise driven rate equations for the photon density S and electron density N are as follows:

$$\frac{dN}{dt} = \frac{I}{eV} - \frac{N}{t_s} - A(N - N_0)S + F_n(t), \qquad (1)$$

$$\frac{dS}{dt} = A(N - N_0)S - \frac{S}{t_p} + \frac{\beta N}{t_s} + F_s(t). \qquad (2)$$

Here I is the injection current, e is the electron charge, V is the volume of the active region, t_s and t_p are the spontaneous electron lifetime and photon lifetime, respectively, β is the spontaneous emission factor, A the gain constant, and N_0 is the minimum electron density required for gain. The noise sources $F_n(t)$ and $F_s(t)$ are delta-correlated Gaussian random variables with zero mean. Their variances can be found in Ref. 9, for example. Assuming a sinusoidal modulation of the injection current, so that $I = I_b + I_m \sin \omega t$, we have obtained numerical solutions of Eqs. (1) and (2). The random variables F_n and F_s are supplied by the computer's random number generator. Figure 3(a) shows a period-doubled solution of the rate equations in the absence of noise for the parameter values $I_b = 1.1 I_{\text{th}}$, $I_m = 0.5 I_b$, and $\omega = 1.3\omega_0$ (ω_0 is the relaxation oscillation frequency). In Fig. 3(b) we show one realization of the same period-two solution in the presence of noise. The inclusions of quantum noise leads to large amplitude fluctuations in the modulated output. These fluctuations inhibit the development of stable period 2^n oscillations for $n > 2$ thus truncating the bifurcation series.

In conclusion, we have observed subharmonic bifurcations of up to period-four in the output of modulated semiconductor lasers. The destabilizing effect of intrinsic quantum noise on higher order bifurcations has also been observed. Intermittent regions of irregular pulsation have been seen which may signal the onset of chaotic behavior.

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